ANALYTICAL METHOD OF SIZING PHOTOVOLTAIC WATER PUMPING SYSTEM

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Abstract—**Photovoltaic Water Pumping System (PWPS) is a well known technology deployed in remote areas for the provision of drinking water and also for irrigation. Due to the high cost of solar energy implementation, the system becomes very costly. Unfortunately, innacuracies in the system sizing mostly lead to oversizing, resulting in huge waste of money. This paper presents a mathematical method of sizing photovoltaic water pumping systems with more accuracy. The method starts with the assessment of losses through pipes and other accessories by using Poiseuil, Blasius and Blench laws. It also considers data on solar irradiation, average temperature as well as necessary parameters on the solar panel itself. It helps to calculate accurately the peak power that must be generated according to a demand. Moreover, simulations have been done in MATLAB to emphasize the effect of neglecting the temperature, the solar irradiation and the pipeline losses on the sizing method. Results show that big variations of temperature influence the sizing negatively, requiring more power than necessary; Bigger Solar irradiation requires fewer peak power. Again, hydraulic losses could add up to 10% of the necessary peak power.**

Index Terms—**Solar Irradiation, Temperature, Pumping system, Peak Power, Hydraulic losses, Simulation.**

I. INTRODUCTION

MORE than a billion people, almost one fifth of the world's population, lacks access to safe drinking water, and 40 percent lacks access to basic sanitation, according to the 2nd UN World Water Development Report [1]. Arid zones are well known for their lack of water. There is a great and urgent need to provide an environmentally technology for the provision of potable water in remote areas [2]. Water pumping system is the key component in meeting this demand.

A Solar-Powered Water Pumping System uses solar energy to power a pump to supply population with water. Solar pumping system sizing and optimisation is a complex task relying on numerous parameters. Most pump manufacturers do indeed propose their own "standard" system configuration or simple tools for a fast sizing, but only applicable on one standard irradiation day. To explain further, traditional methods of sizing PWPS involve the use of graphics given by manufacturers. On the basis of two fundamental parameters (head and flow), the power peak of a PWPS could be read on a graph provided by the manufacturer. However, some other parameters including temperature, solar radiation and others do influence considerably the system sizing. Later, André

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Mermoud developed two projects [3, 4] which help improving the sizing method by considering an average solar irradiation over a minimum period of one year. A wrong sizing of PWPS can either results in a system that could not meet the demand of water or an oversized system leading to unnecessary heavy financial burden. This demotivates a lot of stakeholders in promoting the technology. Therefore the sizing method must be accurately revised in order to determine the effective peak power needed for a certain design while considering all the necessary parameters.

Furthermore, the water pumped by a PWPS is stored in a reservoir. The size of the reservoir is, however, often roughly estimated in conventional design procedures. [5] introduced a design tool for sizing the water tank that corresponds exactly to the need. Other studies on the photovoltaic system help people to easily design their own system. They are mainly guide to design PVP system. For instance [6], guide to solar powered water pumping systems in New-York state by the Energy researches and development authority of the New-York State provides general principles required to make an informed decision on whether or not a solar pump is right; [7], guide to photovoltaic (PV) system design and installation, by the Endecon Engineering also aims at providing tools and guidelines for the installer to help ensure that residential photovoltaic power systems are properly specified and installed, resulting in a system that operates to its design potential;

Reducing the cost of PVP system is then the main target for a lot of projects. In this line, [8] develops a project which deals with the design and simulation of a simple but efficient photovoltaic water pumping system. It provides theoretical studies of photovoltaic and modelling techniques using equivalent electric circuits. The system employs MPPT. PSpice simulations verify the DC-DC converter design. MATLAB simulations perform comparative tests of two popular MPPT algorithms using actual irradiance data. In addition, [4] work on the Modelling and simulation of a pumping system fed by photovoltaic generator within the Matlab/Simulink programming environment.

In this article, we propose a novel method of sizing method to sizing PWPS. The method involves the assessment of losses through the pipe and other accessories by using Poiseuil, Blasius and Blench laws in order to calculate accurately, the hydraulic head. In addition to the head, the method adopts the use of effective temperature and solar radiation accurately calculate the peak power that must be generated according to the demand.

II. SIZING METHOD

The PVP sizing model is done through the following steps: determination of total head, determination of hydraulic energy, determination of electrical energy, determination of available solar energy and finally, determination of electric peak power.

The total head H_T is the sum of the static head, the difference between the static and the dynamic head, the dynamic head itself and the losses. Figure 1 illustrates the various types of head.

$$H_T = H_S + H + H_D + H_d \quad (1)$$

H_S: static head, difference between the surface of the water and the deepest point to which the water must be pumped; H represents the drawdown, the difference between the static and the dynamic level of water; H_D dynamic head represents the losses through the pipeline; H_d: singular losses (in a specific component)
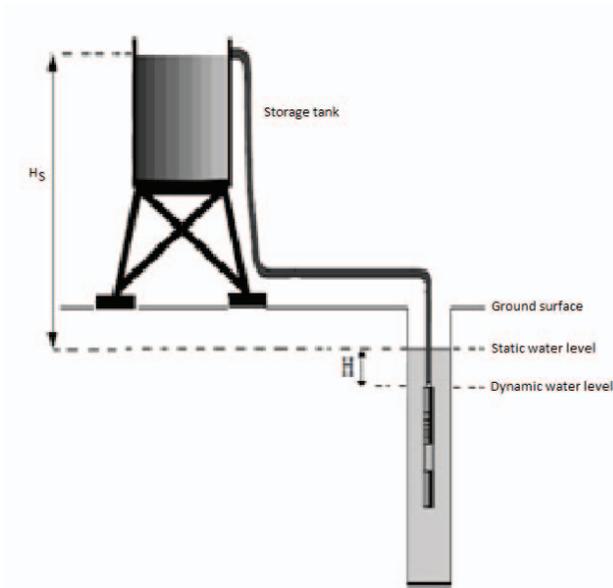

Figure 1.   Adopted model for the calculation of hydraulic head

Next section discusses the calculation of each of the parameters involved in the total head calculation.

Load losses (H_D) in a pipeline depend on the following elements: the pipeline length L, the water viscosity μ, the internal diameter of the pipeline D, the flow Q and the pipeline roughness ε. Table 1 [10] gives some values of absolute roughness depending on the material in direct contact with the fluid.

Table I
ABSOLUTE ROUGHNESS OF SOME MATERIALS

Material	Absolute Roughness
Stainless steel	1.8×10^{-3}
PVC	0.06×10^{-3}
Aluminium	0.06×10^{-3}
Galvanized Iron	6×10^{-3}
Fiber glass	0.02×10^{-3}
Plastic	0.06×10^{-3}
Ductile iron	102

The Darcy-Weisbach formula helps to calculate the losses in the pipeline:

$$H_D = \lambda \cdot \frac{L}{D} \cdot \frac{V^2}{2g} \quad (2)$$

Where: L is the pipeline length (m), D, the pipeline internal diameter (m), g, the gravity acceleration (m/s2), V the average speed of the water in (m/s) and λ the friction coefficient of the pipeline.

The average speed V is related to the average water flow Q by the equation

$$Q = S \times V \quad (3)$$

with S, the cross-sectional area of the pipeline.

$$S = \frac{D^2}{4} \cdot \pi \quad (4)$$

Then, flow Q becomes:

$$Q = \frac{D^2}{4} \times \pi \times V \quad (5)$$

V can be deduced:

$$V = \frac{4}{\pi \times D^2} \times Q \quad (6)$$

with V in m/s and Q in m^3/s

The determination of the friction coefficient λ involved in the calculation of H_D relies on the flow of water. Fluid flows are classified into three categories: laminar flow, moderate turbulent flow and rough turbulent flow. Reynolds criterion helps to make a quantitative difference between the three types of flow [10] by using a criteria based on the Reynolds number R_e .

$$R_e = \frac{4 \times \rho \times Q}{\pi \times \mu \times D} \quad (7)$$

where ρ, Q, D and μ are respectively, water density, water flow, diameter of the pipeline and dynamic viscosity. Table 2 [11] shows some values of dynamic viscosity:

Table II
WATER DYNAMIC VISCOSITY VALUES UNDER DIFFERENT TEMPERATURES

Temperature	Dynamic viscosity
0	1791.5
5	1519.3
10	1307.0
15	1138.3
20	1002.0
25	890.2
30	797.3
35	719.1
40	652.7
50	547.1
60	467
70	404.6
80	355.1
90	315
100	282.1

After calculating the Reynolds number, the condition below must be checked in order to select the best formula that calculates the friction coefficient.

laminar flow: $R_e < 2000$. The formula is given by Poiseuil law: $\lambda = 64/R_e$

Moderate turbulent flow: $2000 < R_e < 10000$. The formula is given by Blasius law: $0.316 \times e^{-0.25}$

Rough turbulent flow: $R_e > 10000$. The formula is given by Blench Law: $\lambda = 0.79 \times \sqrt{\frac{\varepsilon}{D}}$

ε is the roughness. The relative roughness is defined as the ratio between the absolute roughness and the internal diameter of the pipe. $\varepsilon_r = \frac{\varepsilon}{D}$

Once the friction coefficient known, the next step is to calculate the singular losses H_d.

Whenever tools like valve, elbow, junctions are added to the pipeline, they bring extra load losses which can be evaluated with the formula [11]:

$$H_d = K_{ac} \cdot \frac{V^2}{2 \cdot g} \tag{8}$$

where K_{ac} is a coefficient related to component type. Table 3 [5] shows K_{ac} values for different accessories:

Table III
FRICTION COEFFICIENT VALUES

Tools	Coefficient K_{ac}
Junction from the tank to the pipeline	0.5
Junction from the pipeline to the tank	0.1
Elbow 45°	0.35 - 0.45
Elbow 90°	0.5 -0.75
Object which have the form of T	1.5 - 2
Control valve (open)	3

Considering formulae 1, 2 and 8, the total head can be evaluated by:

$$H_T = H_S + H + \lambda \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \sum K_{ac} \cdot \frac{V^2}{2 \cdot g} \tag{9}$$

Thus

$$H_T = H_S + H + \frac{V^2}{2 \cdot g} \cdot \left(\lambda \cdot \frac{L}{D} + \sum K_{ac} \right) \tag{10}$$

Knowing the total head in addition to the required volume of water for a determined period and the characteristics of the borehole, the necessary average hydraulic energy on the same period is calculated with the formula:

$$E_H = \frac{\rho \cdot g \cdot Q \cdot H_T}{3600} \tag{11}$$

Where: E_H is the hydraulic energy per day (Wh/day), H_T is the total head (m) ρ is the water density (1000 kg/m3), g is the acceleration due to gravity (9,81m/s2) and Q, the amount of water pumped per day (m3/day). By considering:

$$C_H = \frac{\rho \cdot g}{3600} \tag{12}$$

The hydraulic energy becomes:

$$E_H = C_H \times Q \times H_T \tag{13}$$

The next step is to calculate the power peak of the solar array. Accordng to A.Hadj Arab (2005), [12] the output power of a photovoltaic generator under standard conditions

of measurements which are lighting $G_{ce} = 1000 \ W/m^2$ and cell temperature $T_{cref} = 25^oC$, is given by formula 14:

$$E_e = \eta_g \cdot A \cdot G_{dm} \tag{14}$$

where: P_p is the output power (W), η_g is the generator efficiency at the reference temperature, A is the active surface of the generator (m^2), G_{ce}: lighting ($1000 W/m^2$) The daily electric energy is given by:

$$E_e = \eta_{PV} \cdot A \cdot G_{dm} \tag{15}$$

where: η_{PV} is the daily average efficiency of the generator in exploitation conditions, G_{dm} is the daily average irradiation on the module plan ($Wh/m^2/day$).

The efficiency η_{PV} can be calculated by the following expression:

$$\eta_{PV} = F_m \cdot [1 - \gamma \cdot (T_c - T_{cref}) \cdot \eta_g] \tag{16}$$

Where γ is the temperature coefficient of the cell; For mono-silicon module, γ varies from 0.004 to 0.005 /°C. For poly-silicon module, γ varies from 0.001 to 0.002/°C. Tc is the daily average temperature of cell during hot time.

The necessary electric energy is related to the hydraulic energy by the following expression

$$E_e = \frac{E_H}{\eta_{PV}} \tag{17}$$

Where: Eh is the monthly average hydraulic energy (Wh), η_{PV} is the efficiency of the sub-system pump-motor.

Recalling Equation 15:

$$A = \frac{E_e}{\eta_{PV} \cdot G_{dm}} \tag{18}$$

By replacing η_{PV} and A in (14) by their respective equivalent (16) and (18), we obtain:

$$P_p = \frac{G_{ce}}{F_m \cdot [1 - \gamma \cdot (T_c - T_{cref}) \cdot \eta_g] \cdot G_{dm}} \cdot \frac{E_H}{\eta_{PV}} \tag{19}$$

Equation (13) gives the expression of E_h which can be inserted in the formula as follow:

$$P_p = \left(\frac{G_{ce}}{F_m \cdot [1 - \gamma \cdot (T_c - T_{cref}) \cdot \eta_g] \cdot G_{dm}} \cdot \frac{C_H}{\eta_{PV}} \right) \cdot Q \cdot H_T \tag{20}$$

Assuming a constant C_p defined by:

$$C_p = \frac{G_{ce}}{F_m \cdot [1 - \gamma \cdot (T_c - T_{cref}) \cdot \eta_g] \cdot G_{dm}} \cdot \frac{C_H}{\eta_{PV}} \tag{21}$$

The power expression becomes:

$$P_p = C_p \cdot Q \cdot H_T \tag{22}$$

III. RESULTS

In this section, we simulate with MATLAB, the effect of the variation of total head, the temperature and the solar irradiation on the peak power.

For an average temperature $T = 27^oC$ and Solar irradiation $Gdm = 4KWh/m^2$, figure 2 shows the variation of power versus hydraulic head for five different values of water flow.

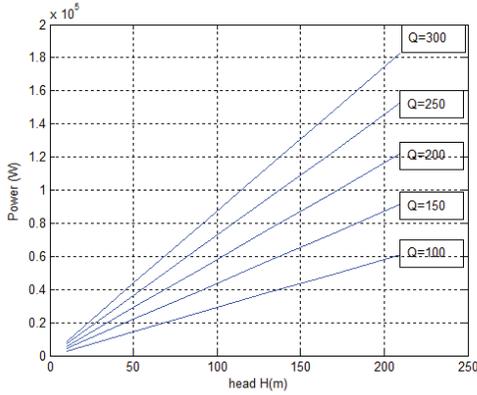Figure 2.   Peak power variation vs total head for five different values of water flow Q

The power increases proportionally to the hydraulic head for a constant water flow. As the water flow increases, the peak power also increases. A neglect of the hydraulic losses calculated in our proposed method will bring a reduction on the effective hydraulic head. For an effective head of $100m$ with 10% losses at a flow of $200m^3/s$, the error in sizing, using figure 2, can be evaluated to $E = 6000 - 5000 = 1000W$. This will result in a system that could not meet the requirements.

Furthermore we investigate the effect of solar irradiation and temperature on the power peak respectively in figure 3 and figure 4. These two factors are related to climate conditions.
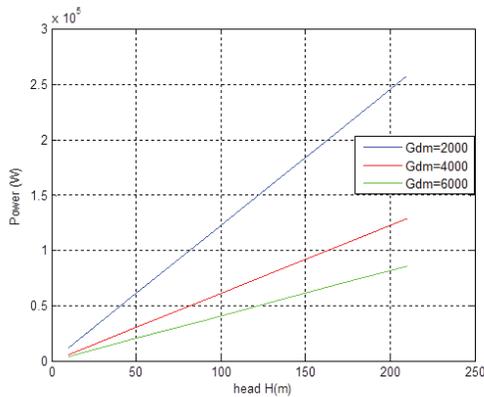

Figure 3.   Effect of Solar Irradiance variation on the peak power

The curves shows that a big variation of temperature has a negative effect on the solar panel leading to lower power production. Therefore, the power peak at the implementation stage must be increased to avoid system under-sized.
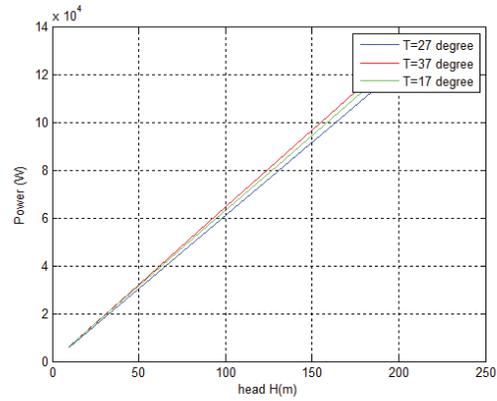

Figure 4.   Effect of Temperature variation on the peak power

Finally, an increase solar radiation positively affects the system. Under good solar irradiation the PWPS need lower power to satisfy the same demand.

IV. CONCLUSIONS

This paper proposed an analytical method of sizing photovoltaic water pumping system. The method took in consideration the calculation of hydraulic losses, the effect of temperature and solar irradiation. Simulation results obtain with MATLAB shows that unlike the traditional sizing method based on graphs, the adoption of the proposed method could easily help to avoid both under-sizing and over-sizing of the system. Moreover, temperature variation negatively affect the sizing leading to higher implementation cost while an increase in solar irradiation does the opposite. However, due to the complexity of the proposed method, we recommend that a software should be built on it to handle the calculations easily.

NOMENCLATURE

A active area of the generator (m^2)
H difference between static and dynamic level (m)
H_s static head (m)
H_T Total head (m)
H_D Load losses through pipeline (m)
H_d singular load losses (m)
L pipeline length (m)
D pipeline internal diameter (m)
V average speed of the water (m/s)
λ friction coefficient of the pipeline
g gravity acceleration $(9.81m/s^2)$
Q water flow(amount of water pumped per day) m^3/day
μ Water viscosity
ε roughness of the pipeline
ε_r relative roughness
K_{ac} friction coefficient related to singular component or tool.
R_e Reynolds number
G_{ce} lighting $(1000W/m^2)$
G_{dm} daily average irradiation $(Wh/m^2/day)$.
E_H hydraulic energy per day
Ee Daily electrical energy (Wh/day)
P_p output power (W)
ρ water density (1000 kg/m3)
γ temperature coefficient of the photovoltaic cell;
T_c daily average temperature of cell during hot time

η_g generator efficiency at the reference temperature

η_{PV} daily average efficiency of the generator in exploitation conditions,

η_{MP} efficiency of the sub-system motor pump.

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