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# Twisted Yang–Mills field theory: connections and Noether currents

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## Abstract

Main properties of noncommutative (NC) gauge theory are investigated in a two-dimensional twisted Moyal plane, generated by vector fields  $X_a = e_a^\mu(x)\partial_\mu$ ; the dynamical effects are induced by a nontrivial tensor  $e_a^\mu(x)$ . Connections in such a NC space are defined. Symmetry analysis is performed and related NC action is proved to be invariant under defined NC gauge transformations. A locally conserved Noether current is explicitly computed. Both commuting and noncommutative vector fields  $X_a$  are considered.

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## 1. Introduction

The construction of noncommutative (NC) field theories in a nontrivial background metric generally implies a non-constant deformation matrix  $\tilde{\Theta}^{\mu\nu} = \tilde{\Theta}^{\mu\nu}(x)$ , which naturally results in the difficulty of finding a suitable explicit closed Moyal-type formula, and consequently, defining a NC product becomes rather complicated. The situation is simpler when one deals with the Moyal space  $\mathbb{R}_\Theta^D$ , i.e. the deformed  $D$ -dimensional space endowed with a constant Moyal  $\star$ -bracket of coordinate functions  $[x^\mu, x^\nu]_\star = i\Theta^{\mu\nu}$ . In this case, the star product (see [1–16] and reference therein) is defined by

$$(f \star g)(x) = m\left\{e^{i\frac{\Theta^{\mu\nu}}{2}\partial_\mu \otimes \partial_\nu} f(x) \otimes g(x)\right\} \quad x \in \mathbb{R}_\Theta^D \quad \forall f, g \in C^\infty(\mathbb{R}_\Theta^D), \quad (1)$$

where  $m$  is the ordinary multiplication of functions, i.e.  $m(f \otimes g) = f.g$ . In the coordinate basis, this space is generated by the usual commuting vector field  $\partial_\mu =: \frac{\partial}{\partial x^\mu} \in T_x \mathbb{R}_\Theta^D$ , the tangent space of  $\mathbb{R}_\Theta^D$ , conferring to Moyal space the properties of a flat space.

In contrast, in the context of a dynamical NC field theory, the vector field can be generalized to take the form  $X_a = e_a^\mu(x)\partial_\mu$ , where  $e_a^\mu(x)$  is a tensor depending on the coordinate functions

in the complex general linear matrix group of order  $D$ ,  $GL(D, \mathbb{C})$ . The star product then takes the form

$$(f \star g)(x) = m \left\{ e^{i \frac{\Theta^{ab}}{2} X_a \otimes X_b} f(x) \otimes g(x) \right\} \quad x \in \mathbb{R}_{\Theta}^D \quad \forall f, g \in C^\infty(\mathbb{R}_{\Theta}^D) \quad (2)$$

and the vielbeins are given by the infinitesimal affine transformation as

$$e_a^\mu(x) = \delta_a^\mu + \omega_{ab}^\mu x^b, \quad (3)$$

where  $\omega_{ab} \in GL(D, \mathbb{C})$ . Using (3), the non-vanishing Lie bracket peculiar to the non-coordinate base [7]

$$[X_a, X_b] = e^c [e_a^\mu \partial_\mu e_b^\nu - e_b^\mu \partial_\mu e_a^\nu] X_c = C_{ab}^c X_c \quad (4)$$

is here simply reduced to

$$[X_a, X_b] = \omega_{ba}^\mu \partial_\mu - \omega_{ab}^\mu \partial_\mu = -2\omega_{ab}^\mu \partial_\mu. \quad (5)$$

Besides, the dynamical star product (2) can now be expressed as

$$(f \star g)(x) = m \left[ \exp \left( \frac{i}{2} \theta e^{-1} \epsilon^{\mu\nu} \partial_\mu \otimes \partial_\nu \right) (f \otimes g)(x) \right] \quad (6)$$

where  $e^{-1} =: \det(e_a^\mu) = 1 + \omega_{12}^1 x^2 - \omega_{12}^2 x^1$ ;  $\epsilon^{\mu\nu}$  is the symplectic tensor in two dimensions ( $D = 2$ ), i.e  $\epsilon^{12} = -\epsilon^{21} = 1$  and  $\epsilon^{11} = \epsilon^{22} = 0$ .

The coordinate function commutation relation becomes  $[x^\mu, x^\nu]_\star = i \tilde{\Theta}^{\mu\nu} = i(\Theta^{\mu\nu} - \Theta^{a[\mu} \omega_{ab}^{\nu]} x^b)$  which can be reduced to the usual Moyal space relation, as expected, by setting  $\omega_{ab}^\mu = [0]$ . One can check that the Jacobi identity is also well satisfied, i.e.

$$[x^\mu, [x^\nu, x^\rho]_\star]_\star + [x^\rho, [x^\mu, x^\nu]_\star]_\star + [x^\nu, [x^\rho, x^\mu]_\star]_\star = \Theta^{b\mu} \Theta^{d[\nu} \omega_{bd}^{\rho]} = 0 \quad (7)$$

conferring a Lie algebra structure to the defined twisted Moyal space. This identity ensures the associativity of the star product (2) and implies that

$$\tilde{\Theta}^{\sigma\rho} \partial_\rho \tilde{\Theta}^{\mu\nu} + \tilde{\Theta}^{\nu\rho} \partial_\rho \tilde{\Theta}^{\sigma\mu} + \tilde{\Theta}^{\mu\rho} \partial_\rho \tilde{\Theta}^{\nu\sigma} = 0. \quad (8)$$

Remark that with relation (5), the requirement that  $\omega_{ab}$  is a symmetric tensor trivially ensures the associativity of the star product. In the interesting particular case addressed in this work, the associativity of the star product (2) can be shown even for the non-symmetric tensor  $\omega_{ab}$ . See the [appendix](#) for details.

The particular condition  $[X_a, X_b] = 0$  (i.e. the vector fields are commuting), results in constraints on  $e_a^\mu$ , namely  $e_{[a}^\mu \partial_\mu e_{b]}^\nu = 0$ , that can be solved off-shell in terms of  $D$  scalar fields  $\phi^a$ . Supposing that the square matrix  $e_a^\mu$  has an inverse  $e_\mu^a$  everywhere so that the  $X_a$  are linearly independent, then the above condition becomes  $\partial_{[\mu} e_{\nu]}^a = 0$  which is satisfied by  $e_\nu^a = \partial_\nu \phi^a$ . Since  $X_a \phi^b = \delta_a^b$ , the field  $\phi^b$  can be viewed as new coordinates along the  $X_a$  directions. The metric  $g$  on  $\mathbb{R}_{\Theta}^D$  can be chosen to be  $g(X_a, X_b) = e_a^\mu e_b^\nu g_{\mu\nu} = \delta_{ab}$ . See [3–6] for more details. In the whole work, we deal with Euclidean signature and  $D = 2$ .

The paper is organized as follows. In section 2, we provide the general properties of gauge theory in twisted Moyal space and define related connections. The tensor  $\omega_{ab}^\mu$  is an infinitesimal tensor, skew-symmetric in the indices  $a$  and  $b$ . In section 3, we study the symmetry of pure gauge theory and show that the related NC action is invariant under  $U_\star(1)$  gauge transformation. Besides, we compute the resulting Noether current. In section 4, we investigate the properties of the model for commuting vector fields  $X_a$ . Section 5 is devoted to concluding remarks.

## 2. Connections and gauge transformation

Consider  $E = \{x^\mu, \mu \in \{1, 2\}\}$  and  $\mathbb{C}[[x^1, x^2]]$ , the free algebra generated by  $E$ . Let  $\mathcal{I}$  be the ideal of  $\mathbb{C}[[x^1, x^2]]$ , engendered by the elements  $x^\mu x^\nu - x^\nu x^\mu - i\tilde{\Theta}^{\mu\nu}$ . The twisted Moyal algebra  $\mathcal{A}_{\tilde{\Theta}}$  is the quotient  $\mathbb{C}[[x^1, x^2]]/\mathcal{I}$ . Each element in  $\mathcal{A}_{\tilde{\Theta}}$  is a formal power series in the  $x^\mu$ 's for which the relation  $[x^\mu, x^\nu]_\star = i\tilde{\Theta}^{\mu\nu}$  holds. Here, Moyal algebra can also be defined as the linear space of smooth and rapidly decreasing functions equipped with the NC star product given in (2). The gauge symmetries on this NC space can be realized in their enveloping algebra. However, there is an isomorphism mapping the NC function algebra  $\mathcal{A}_{\tilde{\Theta}}$  into the commutative one, equipped with an additional NC  $\star$ -product [1]. We consider the following infinitesimal affine transformation:

$$e_a^\mu(x) = \delta_a^\mu + \omega_{ab}^\mu x^b, \quad \omega_{ab}^\mu =: -\omega_{ba}^\mu \quad \text{and} \quad |\omega^\mu| \ll 1. \quad (9)$$

For  $D = 2$ ,  $e_a^\mu$  and  $\Theta^{ab}$  can be expressed as follows:

$$(e)_a^\mu = \begin{pmatrix} 1 + \omega_{12}^1 x^2 & \omega_{12}^2 x^2 \\ -\omega_{12}^1 x^1 & 1 - \omega_{12}^2 x^1 \end{pmatrix} \quad \text{and} \quad (\Theta)^{ab} = \begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix} = \theta(\epsilon)^{ab}, \quad (10)$$

where  $\epsilon^{12} = -\epsilon^{21} = 1$  and  $\epsilon^{11} = \epsilon^{22} = 0$ . There follow the relations

$$e^{-1} =: \det(e_a^\mu) = 1 + \omega_{12}^1 x^2 - \omega_{12}^2 x^1 \quad (11)$$

$$e =: \det(e_\mu^a) = 1 - \omega_{12}^1 x^2 + \omega_{12}^2 x^1. \quad (12)$$

The tensor  $\tilde{\Theta}^{\mu\nu}$  can be written as [7]

$$(\tilde{\Theta})^{\mu\nu} = (\Theta)^{\mu\nu} - (\Theta^{a[\mu} \omega_{ab}^{v]}) x^b = \begin{pmatrix} 0 & \theta e^{-1} \\ -\theta e^{-1} & 0 \end{pmatrix}. \quad (13)$$

Let us now define the spacetime ( $M \subseteq \mathbb{R}_{\tilde{\Theta}}^2$ ) metric as

$$\mathfrak{g}_{\mu\nu} = e_\mu^a e_\nu^b \delta_{ab} = \begin{pmatrix} 1 - 2\omega_{12}^1 x^2 & \omega_{12}^1 x^1 - \omega_{12}^2 x^2 \\ \omega_{12}^1 x^1 - \omega_{12}^2 x^2 & 1 + 2\omega_{12}^2 x^1 \end{pmatrix}, \quad (14)$$

$$\text{where } e_\mu^a = \begin{pmatrix} 1 - \omega_{12}^1 x^2 & -\omega_{12}^2 x^2 \\ \omega_{12}^1 x^1 & 1 + \omega_{12}^2 x^1 \end{pmatrix}, \quad (15)$$

and its inverse as

$$\mathfrak{g}^{\mu\nu} = e_a^\mu e_b^\nu \delta^{ab} = \begin{pmatrix} 1 + 2\omega_{12}^1 x^2 & \omega_{12}^2 x^2 - \omega_{12}^1 x^1 \\ \omega_{12}^2 x^2 - \omega_{12}^1 x^1 & 1 - 2\omega_{12}^2 x^1 \end{pmatrix}, \quad (16)$$

$$\text{where } e_a^\mu = \begin{pmatrix} 1 + \omega_{12}^1 x^2 & \omega_{12}^2 x^2 \\ -\omega_{12}^1 x^1 & 1 - \omega_{12}^2 x^1 \end{pmatrix}, \quad (17)$$

with  $g = -\det(\mathfrak{g}_{\mu\nu})$ . NC field theory over Moyal algebra of functions can be defined as field theories over the module  $\mathcal{H}$  on the NC algebra  $\mathcal{A}_{\tilde{\Theta}}$  or as matrix theories with coefficients in  $\mathcal{A}_{\tilde{\Theta}}$ . In the following, we restrict the study of field modules to rank trivial bi-modules  $\mathcal{H}$  over  $\mathcal{A}_{\tilde{\Theta}}$  with a Hilbert space structure defined by the scalar product

$$\langle a, b \rangle =: \int e d^2x \text{Tr}(a^\dagger \star b) \star e^{-1}; \quad a, b \in \mathcal{A}_{\tilde{\Theta}}. \quad (18)$$

Provided this framework, the notion of connection defined on vector bundles in ordinary differential geometry is replaced, in NC geometry, by the generalized concept of connection on the projective modules as follows.

**Definition 2.1.** The sesquilinear maps  $\nabla_\mu : \mathcal{H} \rightarrow \mathcal{H}$  are called connections if they satisfy the differentiation chain rule

$$\nabla_\mu(m \star f) = m \star \partial_\mu(f) + \nabla_\mu(m) \star f \quad \text{for } f \in \mathcal{A}_{\tilde{\Theta}} \quad \text{and} \quad m \in \mathcal{H} \quad (19)$$

(assumed here to be a right module over  $\mathcal{A}_{\tilde{\Theta}}$ ), and if they are compatible with the Hermitian structure of  $\mathcal{H}$  defined as  $h(f, g) = f^\dagger \star g$ , i.e.

$$\partial_\mu h(m_1, m_2) = h(\nabla_\mu m_1, m_2) + h(m_1, \nabla_\mu m_2). \quad (20)$$

In the following, we can identify  $\mathcal{A}_{\tilde{\Theta}}$  with  $\mathcal{H}$ .

**Definition 2.2.** Denoting by  $\mathbf{1}$  the unit element of  $\mathcal{A}_{\tilde{\Theta}}$ , we define the gauge potential by  $\nabla_\mu \mathbf{1} = -iA_\mu$ . Then the connection can be explicitly written as

$$\nabla_\mu(\cdot) = \partial_\mu(\cdot) - iA_\mu \star (\cdot). \quad (21)$$

$A_\mu$  is called the gauge potential in the fundamental representation.

Note that the left module can be used to define the connection in the anti-fundamental representation by  $\nabla_\mu(\cdot) = \partial_\mu(\cdot) + i(\cdot) \star A_\mu$ . In the same vein, the module can be used to define the connection on the adjoint representation by  $\nabla_\mu(\cdot) = \partial_\mu(\cdot) - i[A_\mu, (\cdot)]_\star$ . Here, we adopt the fundamental representation. Now, we define the gauge transformation as a morphism of module, denoted by  $\gamma$ , satisfying the relation

$$\gamma(m \star f) = \gamma(m) \star f \quad \text{for } f \in \mathcal{A}_{\tilde{\Theta}} \quad \text{and} \quad m \in \mathcal{H} \quad (22)$$

and preserving the Hermitian structure  $h$ , i.e.

$$h(\gamma(f), \gamma(g)) = h(f, g) \quad \text{for } f, g \in \mathcal{A}_{\tilde{\Theta}}. \quad (23)$$

For  $f = g = \mathbf{1}$ , one can prove that  $\gamma(\mathbf{1}) \in U(\mathcal{A}_{\tilde{\Theta}})$ , the group of unitary elements of  $\mathcal{A}_{\tilde{\Theta}}$ , i.e.  $\gamma^\dagger(\mathbf{1}) \star \gamma(\mathbf{1}) = \mathbf{1}$ .

Note that the Jacobi identity is covariantly written in the form

$$\tilde{\Theta}^{\sigma\rho} \nabla_\rho \tilde{\Theta}^{\mu\nu} + \tilde{\Theta}^{\nu\rho} \nabla_\rho \tilde{\Theta}^{\sigma\mu} + \tilde{\Theta}^{\mu\rho} \nabla_\rho \tilde{\Theta}^{\nu\sigma} = 0. \quad (24)$$

This equation is evidently satisfied whenever the following condition holds:

$$\nabla_\rho \tilde{\Theta}^{\mu\nu} = 0, \quad (25)$$

which is very simple to handle in two-dimensional space. Indeed, in  $D = 2$  the most general  $\tilde{\Theta}$  can be written in the form

$$\tilde{\Theta}^{\mu\nu} = \frac{\epsilon^{\mu\nu}}{\sqrt{-g(x)}} \theta(x^1, x^2), \quad (26)$$

where  $\theta(x^1, x^2)$  is a constant, simply denoted by  $\theta$ . Then

$$e^{-1} = 1/\sqrt{-g(x)} \quad \text{or} \quad e = \sqrt{-g(x)}. \quad (27)$$

To end this section, let us mention that the integral  $\int d^D x f \star g$ , defined with the dynamical Moyal  $\star$ -product (2), is not cyclic, even with suitable boundary conditions at infinity, i.e.

$$\int d^D x (f \star g) \neq \int d^D x (g \star f). \quad (28)$$

Using now the measure  $e d^D x$  where  $e = \det(e_\mu^a)$ , a cyclic integral can be defined so that, up to boundary terms,

$$\int e d^D x (f \star g) = \int e d^D x (fg) = \int e d^D x (g \star f). \quad (29)$$

In flat space,  $\sqrt{-g(x)} = 1$ .

### 3. Dynamical pure gauge theory

We consider a field  $\psi$ , an element of the algebra  $\mathcal{A}_{\tilde{\Theta}}$  ( $\psi \in \mathcal{A}_{\tilde{\Theta}}$ ), and its infinitesimal gauge variation  $\delta\psi$  such that, under an infinitesimal gauge transformation  $\alpha(x)$ , the relation  $\delta_\alpha\psi(x) = i\alpha(x) \star \psi(x)$  is obeyed. The covariant coordinates are defined as

$$X^\mu = x^\mu + A^\mu, \quad A^\mu \in \mathcal{A}_{\tilde{\Theta}}; \quad (30)$$

$A^\mu$  is called the gauge potential and satisfies the relation  $\delta_\alpha A^\mu = \tilde{\Theta}^{\mu\rho} \partial_\rho \alpha + i[\alpha, A^\mu]_\star$ . One can check that

$$[\alpha(x), \tilde{\Theta}^{\mu\sigma}]_\star = i\Theta^{a[\mu} \omega_{ac}^{\sigma]} \Theta^{c\rho} \partial_\rho \alpha(x) \quad \text{and} \quad \tilde{\Theta}_{\mu\sigma}^{-1} \delta_\alpha \tilde{\Theta}^{\mu\sigma} = 2\omega_{ac}^a \delta x^c. \quad (31)$$

From the last two equations and the definition of  $A_\sigma$  such that  $A^\mu = \tilde{\Theta}^{\mu\sigma} A_\sigma$ , we derived the gauge variation

$$\delta_\alpha A_\sigma = \partial_\sigma \alpha(x) + i[\alpha(x), A_\sigma]_\star + 2\omega_{ac}^a (\Theta^{c\rho} \partial_\rho \alpha(x) - \delta x^c) A_\sigma, \quad (32)$$

which results in two contributions in the expression of  $\delta_\alpha A_\sigma$ : the first contribution consisting of the first two terms of the ordinary Moyal product [2], and the second one given by the last term pertaining to the twisted effects of the theory. Of course, when  $\omega = 0$ , we recover the usual Moyal result.

The NC gauge tensor  $T^{\mu\nu} \in \mathcal{A}_{\tilde{\Theta}}$  is defined by  $T^{\mu\nu} = [X^\mu, X^\nu]_\star - i\tilde{\Theta}^{\mu\nu}$  and satisfies the properties

$$\delta_\alpha T^{\mu\nu} = i[\alpha(x), T^{\mu\nu}]_\star. \quad (33)$$

It is then convenient to use the relation  $T^{\mu\nu} = i\tilde{\Theta}^{\mu\sigma} \tilde{\Theta}^{\nu\lambda} F_{\sigma\lambda}$  to derive the twisted Faraday tensor  $F_{\sigma\lambda}$  as

$$F_{\sigma\lambda} = \partial_\sigma A_\lambda - \partial_\lambda A_\sigma - i[A_\sigma, A_\lambda]_\star - \Theta_{\nu\lambda}^{-1} \Theta^{a[\nu} \omega_{a\sigma}^{\lambda]} A_\lambda + \Theta_{\mu\sigma}^{-1} \Theta^{a[\mu} \omega_{a\lambda}^{\sigma]} A_\sigma - (\Theta_{\mu\sigma}^{-1} \Theta^{a[\mu} \omega_{ac}^{\sigma]} \Theta^{c\rho} \partial_\rho A_\lambda) A_\sigma + (\Theta_{\nu\lambda}^{-1} \Theta^{a[\nu} \omega_{ac}^{\lambda]} \Theta^{c\rho} \partial_\rho A_\sigma) A_\lambda \quad (34)$$

$$= \partial_\sigma A_\lambda - \partial_\lambda A_\sigma - i[A_\sigma, A_\lambda]_\star + 2\omega_{a[\sigma}^a A_{\lambda]} - 2\omega_{ac}^a \Theta^{c\rho} (\partial_\rho A_{[\sigma}) A_{\lambda]}]. \quad (35)$$

Canceling the ‘*twisted*’ contributions involved in the last two terms on the right-hand side of these relations, we turn back to the usual Moyal product result.

We then arrive at the expression of the dynamical NC pure gauge action defined as

$$S_{\text{YM}} = -\frac{1}{4k^2} \int e \, d^2x (F_{\mu\nu} \star F^{\mu\nu} \star e^{-1}) \quad (36)$$

where  $e =: \det(e_\mu^a)$ .

Now, expanding the dynamical  $\star$ -product (2) of two functions as follows:

$$f \star g = fg + \frac{i}{2} \Theta^{ab} X_a f X_b g + \frac{1}{2!} \left(\frac{i}{2}\right)^2 \Theta^{a_1 b_1} \Theta^{a_2 b_2} (X_{a_1} X_{a_2} f)(X_{b_1} X_{b_2} g) + \dots \equiv e^\Delta(f, g), \quad (37)$$

where powers of the bilinear operator  $\Delta$  are defined as

$$\begin{aligned} \Delta(f, g) &= \frac{i}{2} \Theta^{ab} (X_a f)(X_b g) & \Delta^0(f, g) &= fg \\ \Delta^n(f, g) &= \left(\frac{i}{2}\right)^n \Theta^{a_1 b_1} \dots \Theta^{a_n b_n} (X_{a_1} \dots X_{a_n} f)(X_{b_1} \dots X_{b_n} g), \end{aligned} \quad (38)$$

one can deduce the following rules (straightforwardly generalizing the usual Moyal product identities):

$$f \star g = fg + X_a T(\Delta)(f, \tilde{X}^a g) \quad (39)$$

$$[f, g]_\star = f \star g - g \star f = 2X_a S(\Delta)(f, \tilde{X}^a g) \tag{40}$$

$$\{f, g\}_\star = f \star g + g \star f = 2fg + 2X_a R(\Delta)(f, \tilde{X}^a g) \tag{41}$$

with

$$\begin{aligned} T(\Delta) &= \frac{e^\Delta - 1}{\Delta} & S(\Delta) &= \frac{\sinh(\Delta)}{\Delta} \\ R(\Delta) &= \frac{\cosh(\Delta) - 1}{\Delta} & \text{and} & \quad \tilde{X}^a = \frac{i}{2} \Theta^{ab} X_b. \end{aligned} \tag{42}$$

$S(\Delta)(\cdot, \tilde{X}^a \cdot)$  is a bilinear antisymmetric operator and  $T(\Delta)(f, \tilde{X}^a g) - T(\Delta)(g, \tilde{X}^a f) = 2S(\Delta)(f, \tilde{X}^a g)$ .

Define also the gauge transformation  $U$  by

$$\begin{aligned} U &= e_\star^{i\alpha} = \sum_{k=0}^{\infty} \frac{i^k (\alpha)_\star^k}{k!} \\ &= 1 + i\alpha + (i^2/2!) \alpha \star \alpha + (i^3/3!) \alpha \star \alpha \star \alpha + \dots; \quad \alpha \in C^\infty(\mathbb{R}); \end{aligned} \tag{43}$$

$U_\star(1)$  is the NC gauge group generated by the elements  $U \in U_\star(1)$ . The infinitesimal gauge transformation  $U = 1 + i\alpha(x)$  defined in the NC Moyal space is the same as the ordinary infinitesimal gauge transformation in commutative space. Making the gauge transformation of the tensor  $F_{\mu\nu}$  into  $F_{\mu\nu}^U = U \star F_{\mu\nu} \star U^\dagger$ , then the transformed gauge action yields

$$\begin{aligned} S_{\text{YM}}^U &= -\frac{1}{4\kappa^2} \int e \, d^2x (U \star F_{\mu\nu} \star U^\dagger \star U \star F^{\mu\nu} \star U^\dagger \star e^{-1}) \\ &= -\frac{1}{4\kappa^2} \int e \, d^2x (U \star F_{\mu\nu} \star F^{\mu\nu} \star U^\dagger \star e^{-1}) \\ &= -\frac{1}{4\kappa^2} \int e \, d^2x (F_{\mu\nu} \star F^{\mu\nu} \star U^\dagger \star e^{-1} \star U + 2S(\Delta)(U, \tilde{X}^a (F_{\mu\nu} \star F^{\mu\nu} \star U^\dagger \star e^{-1}))) \end{aligned} \tag{44}$$

**Proposition 3.1.** *Provided the stubborn requirement that the surface terms be vanished, the action (36) is invariant under the global gauge transformation, i.e. setting  $\alpha = \alpha_0 = c^{\text{ste}}$  in (43).*

**Proof.** From the infinitesimal gauge transformation  $U(\alpha) = 1 + i\alpha(x)$ , its conjugate given by  $U^\dagger(\alpha) = 1 - i\alpha(x)$  and the definition  $e^{-1} := 1 + \omega_\mu x^\mu$ , where  $\omega_1 = \omega_{12}^1$  and  $\omega_2 = -\omega_{12}^2$ , we have

$$\begin{aligned} U^\dagger \star e^{-1} &= e^{-1} (1 - i\alpha(x)) + \frac{1}{2} \tilde{\Theta}^{\mu\nu} \partial_\mu \alpha(x) \omega_\nu \\ &\Rightarrow U^\dagger \star e^{-1} \star U = e^{-1} + \tilde{\Theta}^{\mu\nu} \omega_\nu \partial_\mu \alpha(x) = e^{-1} + \Theta^{\mu\nu} \omega_\nu \partial_\mu \alpha(x). \\ U^\dagger \star e^{-1} \star U &= e^{-1} \Rightarrow \Theta^{\mu\nu} \omega_\nu \partial_\mu \alpha(x) = 0 \Rightarrow \alpha(x) = \alpha_0 = c^{\text{ste}}. \quad \square \end{aligned}$$

Furthermore, the following statement is true.

**Proposition 3.2.** *Provided the same requirement of vanishing condition of the surface terms, (i) the action (36) is invariant under the NC group of unitary transformations  $U_\star(1)$  defined by the parameter  $\alpha = \alpha_0 + \epsilon_\sigma x^\sigma$ , where  $\epsilon_\sigma$  is an infinitesimal parameter and  $\alpha_0$  is an arbitrary constant, and (ii) there exists an isomorphism between the NC gauge group induced by (32) and  $U_\star(1)$  group.*

**Proof.** Part (i) is immediate from the previous proof. (ii) Imposing the condition  $\alpha = \alpha_0 + \epsilon_\lambda x^\lambda$ , the NC gauge transformation (32) is reduced to the form

$$\delta_\alpha A_\sigma = \partial_\sigma \alpha - \epsilon_\mu \Theta^{\mu\rho} \partial_\rho A_\sigma = \partial_\rho \left( \delta_\sigma^\rho \frac{\alpha}{2} - \epsilon_\mu \Theta^{\mu\rho} A_\sigma \right) = \partial \Lambda \quad (45)$$

giving rise to the isomorphism

$$f : \partial \Lambda \rightarrow e_\star^{i\Lambda} = 1 + i \left( \frac{\alpha}{2} - \epsilon_\mu \Theta^{\mu\rho} A_\sigma \right) \quad (46)$$

mapping the NC gauge group (32) into the  $U_\star(1)$  group.  $\square$

Therefore, the  $U_\star(1)$  group can be considered as the invariance NC gauge group for the Yang–Mills (YM) action defined in (36). Note that setting  $\epsilon_\mu = 0$ , we recover the global gauge transformation of the usual gauge field theory.

The  $A_\mu$  variation of the action (36) is given by

$$\delta_A \mathcal{S}_{\text{YM}} = -\frac{1}{4\kappa^2} \int d^2x (\delta A_\beta \mathcal{E}_A + \partial_\beta J^\beta), \quad (47)$$

where the equation of motion of the field  $A$  is provided by

$$\begin{aligned} \frac{\delta \mathcal{S}_{\text{YM}}}{\delta A_\beta} = \mathcal{E}_A = & -\partial_\mu (e\{F^{\mu\beta}, e^{-1}\}_\star) + \partial_\nu (e\{F^{\beta\nu}, e^{-1}\}_\star) - ie[A_\nu, \{F^{\beta\nu}, e^{-1}\}_\star]_\star \\ & + ie[A_\mu, \{F^{\mu\beta}, e^{-1}\}_\star]_\star - 2\omega_{ac}^a \Theta^{c\rho} (\partial_\rho (-eA_\nu \{F^{\beta\nu}, e^{-1}\}_\star + eA_\mu \{F^{\mu\beta}, e^{-1}\}_\star)) \\ & - 2e\omega_{ac}^a \Theta^{c\rho} (\partial_\rho A_\mu \{F^{\mu\beta}, e^{-1}\}_\star - \partial_\rho A_\nu \{F^{\beta\nu}, e^{-1}\}_\star) + 2e(\omega_{a\mu}^a \{F^{\mu\beta}, e^{-1}\}_\star \\ & - \omega_{a\nu}^a \{F^{\beta\nu}, e^{-1}\}_\star) = 0 \end{aligned} \quad (48)$$

and the current  $J^\beta$  by

$$\begin{aligned} J^\beta = & -\frac{1}{4\kappa^2} [e\delta A_\nu \{F^{\beta\nu}, e^{-1}\}_\star - e\delta A_\mu \{F^{\mu\beta}, e^{-1}\}_\star - ie e_a^\beta (T(\Delta)(\delta A_{[\mu}, \tilde{X}^a[A_\nu], \{F^{\mu\nu}, e^{-1}\}_\star]_\star) \\ & + T(\Delta)([\delta A_{[\mu}, A_\nu]_\star, \tilde{X}^a \{F^{\mu\nu}, e^{-1}\}_\star] + 2S(\Delta)(A_{[\mu}, \tilde{X}^a(\delta A_\nu] \{F^{\mu\nu}, e^{-1}\}_\star))) \\ & - 2\omega_{ac}^a \Theta^{c\beta} e\delta A_{[\mu} A_\nu] \{F^{\mu\nu}, e^{-1}\}_\star + ee_a^\beta (T(\Delta)(\delta F^{\mu\nu}, \tilde{X}^a \{F^{\mu\nu}, e^{-1}\}_\star) \\ & + 2S(\Delta)(F^{\mu\nu}, \tilde{X}^a \delta F^{\mu\nu} \star e^{-1}))], \end{aligned} \quad (49)$$

where  $[\delta A_{[\mu}, A_\nu]_\star] = \delta A_\mu \star A_\nu - A_\nu \star \delta A_\mu - \delta A_\nu \star A_\mu + A_\mu \star \delta A_\nu$ . Using the property that  $F^{\mu\nu} = -F^{\nu\mu}$  and the fact that the surface terms are canceled, the equation of motion  $\mathcal{E}_A = 0$  and the current  $J^\beta$  can be re-expressed, respectively, as

$$\begin{aligned} \frac{\delta \mathcal{S}_{\text{YM}}}{\delta A_\beta} = \mathcal{E}_A = & -2\partial_\mu (e\{F^{\mu\beta}, e^{-1}\}_\star) - 4\omega_{ac}^a \Theta^{c\rho} \partial_\rho (eA_\mu \{F^{\mu\beta}, e^{-1}\}_\star) \\ & - 4e\omega_{ac}^a \Theta^{c\rho} \partial_\rho A_\mu \{F^{\mu\beta}, e^{-1}\}_\star + 4e\omega_{a\mu}^a \{F^{\mu\beta}, e^{-1}\}_\star = 0 \end{aligned} \quad (50)$$

and

$$J^\beta = \frac{1}{2\kappa^2} (e\delta A_\mu \{F^{\mu\beta}, e^{-1}\}_\star + \omega_{ac}^a \Theta^{c\beta} e\delta A_{[\mu} A_\nu] \{F^{\mu\nu}, e^{-1}\}_\star). \quad (51)$$

Let us now deal with the symmetry analysis and deduce the conserved currents. Performing the following functional variation of fields and coordinate transformation

$$A'_\mu(x) = A_\mu(x) + \delta A_\mu(x), \quad x'^\mu = x^\mu + \epsilon^\mu, \quad \epsilon^\mu = \delta x^\mu = 0, \quad (52)$$

and using  $d^2x' = [1 + \partial_\mu \epsilon^\mu + \mathbf{O}(\epsilon^2)] d^2x = d^2x$  lead to the following variation of the action, to first order in  $\delta A_\mu(x)$  and  $\delta\phi^c(x)$ :

$$\begin{aligned} \delta \mathcal{S}_{\text{YM}} = & \int e d^2x \left\{ \left. \frac{\partial x'}{\partial x} \right|_\star (\mathcal{L}'_{\text{YM}} \star e^{-1}) \right\} - \int e d^2x (\mathcal{L}_{\text{YM}} \star e^{-1}) \\ = & \int d^2x \delta((\mathcal{L}_{\text{YM}} \star e^{-1})e) = \int d^2x \{\delta A_\mu((\mathcal{L}_{\text{YM}} \star e^{-1})e)\}, \end{aligned} \quad (53)$$



where

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4\kappa^2} F^{\mu\nu} \star F_{\mu\nu} \quad \text{and} \quad \mathcal{L}'_{\text{YM}} = -\frac{1}{4\kappa^2} F_U^{\mu\nu} \star F_{\mu\nu}^U. \quad (54)$$

On shell, and integrated on a submanifold  $M \subset \mathbb{R}^2$  with fields non-vanishing at the boundary (so that the total derivative terms do not disappear), we obtain

$$\delta \mathcal{S}_{\text{YM}} = \int_M d^2x \partial_\sigma \mathcal{J}^\sigma = 0. \quad (55)$$

**Proposition 3.3.** *The NC Noether current  $\mathcal{J}^\sigma$  is locally conserved.*

**Proof.** The analysis of the local properties of this tensor requires the useful formulas

$$\begin{aligned} \delta_\alpha A_\mu &= \epsilon_\mu (1 + \Theta^{\rho\sigma} \partial_\rho A_\sigma), & \omega_{ac}^a &= -\omega_c, \\ \partial_\beta e &= -\omega_\beta, & \{F^{\mu\nu}, e^{-1}\}_\star &= 2e^{-1} F^{\mu\nu}. \end{aligned} \quad (56)$$

A straightforward computation gives

$$J^\beta = \frac{\epsilon_\mu}{\kappa^2} (1 + \Theta^{\rho\sigma} \partial_\rho A_\sigma) F^{\mu\beta} \Rightarrow \partial_\beta J^\beta = \frac{\epsilon_\mu}{\kappa^2} (1 + \Theta^{\rho\sigma} \partial_\rho A_\sigma) \partial_\beta F^{\mu\beta}. \quad (57)$$

The equation of motion (50) can be simply re-expressed in the form

$$\partial_\mu F^{\mu\beta} = 2\omega_\mu F^{\mu\beta} - 4\omega_c \Theta^{c\rho} (\partial_\rho A_\mu) F^{\mu\beta} - 2\omega_c \Theta^{c\rho} A_\mu \partial_\rho F^{\mu\beta}. \quad (58)$$

Now using the fact that  $\epsilon\omega = 0$  yields the result.  $\square$

**Remark 3.4.**

- The equation of motion (58) is reduced to  $\partial_\mu F^{\mu\beta} = 0$  in an ordinary Moyal plane.
- The action of gauge theory covariantly coupled with the matter fields defined by

$$\mathcal{S} = \mathcal{S}_{\text{YM}} + \mathcal{S}_M, \quad (59)$$

where

$$\begin{aligned} \mathcal{S}_M &= \int_{\mathbb{R}^2} e d^2x [\bar{\psi}(x) (-i\Gamma^\mu \nabla_\mu + m) \psi(x) + \lambda_1 (\bar{\psi} \star \psi \star \bar{\psi} \star \psi)(x) \\ &\quad + \lambda_2 (\bar{\psi} \star \bar{\psi} \star \psi \star \psi)(x)] \star e^{-1}, \end{aligned} \quad (60)$$

is also invariant under global gauge transformation ( $\delta\psi = i\alpha_0\psi$ ,  $\delta\bar{\psi} = -i\alpha_0\bar{\psi}$ ). The current can also be easily deduced in the same manner as above.

#### 4. Case of commuting vector fields

Consider the non-coordinates' base  $e_a^\mu = \delta_a^\mu + \omega_{ab}^\mu x^b$  and the symmetric tensor (between the index  $a$  and  $b$ )  $\omega_{ab}^\mu$ . Then, the twisted star product is naturally associative since

$$[X_a, X_b] = \omega_{ba}^\mu \partial_\mu - \omega_{ab}^\mu \partial_\mu = 0. \quad (61)$$

The matrix representation of  $e_a^\mu$  is given by

$$(e)_a^\mu = \begin{pmatrix} 1 + \omega_{11}^1 x^1 + \omega_{12}^1 x^2 & \omega_{11}^2 x^1 + \omega_{12}^2 x^2 \\ \omega_{12}^1 x^1 + \omega_{22}^1 x^2 & 1 + \omega_{12}^2 x^1 + \omega_{22}^2 x^2 \end{pmatrix} \quad (62)$$

and

$$(e)_\mu^a = \begin{pmatrix} 1 - \omega_{11}^1 x^1 - \omega_{12}^1 x^2 & -\omega_{11}^2 x^1 - \omega_{12}^2 x^2 \\ -\omega_{12}^1 x^1 - \omega_{22}^1 x^2 & 1 - \omega_{12}^2 x^1 - \omega_{22}^2 x^2 \end{pmatrix}. \quad (63)$$

Further,

$$\begin{aligned} e^{-1} &= \det(e_a^\mu) = 1 + (\omega_{11}^1 + \omega_{12}^2)x^1 + (\omega_{22}^2 + \omega_{12}^1)x^2 \\ e &= \det(e_\mu^a) = 1 - (\omega_{11}^1 + \omega_{12}^2)x^1 - (\omega_{22}^2 + \omega_{12}^1)x^2. \end{aligned} \quad (64)$$

The NC tensor is provided by  $(\tilde{\Theta})^{\mu\nu} = \theta e^{-1} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ . Besides, the matrix  $e_\mu^a$  can be written as  $e_\mu^a = \delta_\mu^a + \omega_\mu^{ab} x_b$ , where  $\omega_\mu^{ab} = -\omega_{ab}^\mu$ . Finally, the solution of the field equation  $e_\mu^a = \partial_\mu \phi^a$  is well given by  $\phi^a = x^a + \frac{1}{2} \omega_\mu^{ab} x_b x^v \delta_v^\mu$  as deduced in [7]. The  $\phi^c$  variation of the action can easily be computed and the resulting equation of motion is

$$\frac{\delta \mathcal{S}_{\text{YM}}}{\delta \phi^c} = \mathcal{E}_{\phi^c, A} = e^{-1} X_a (F_{\mu\nu} \star F^{\mu\nu}) - (X_a F^{\mu\nu}) \{F_{\mu\nu}, e^{-1}\}_\star = 0. \quad (65)$$

This variation generates the current

$$\begin{aligned} \mathcal{K}^\beta &= -\frac{e e_b^\beta}{4\kappa^2} [(-F_{\mu\nu} \star F^{\mu\nu} \star \delta\phi^b e^{-1}) + T(\Delta)(X_a (F_{\mu\nu} \star F^{\mu\nu}), \tilde{X}^b (\delta\phi^a e^{-1})) \\ &\quad - T(\Delta)(\delta\phi^a (X_a F_{\mu\nu}), \tilde{X}^b \{F^{\mu\nu}, e^{-1}\}_\star) + \delta\phi^b (F_{\mu\nu} \star F^{\mu\nu} \star e^{-1})] \\ &\quad + 2S(\Delta)(\delta\phi^a (X_a F_{\mu\nu}) \star e^{-1}, \tilde{X}^b F^{\mu\nu}). \end{aligned} \quad (66)$$

Performing the transformation  $\phi'^c(x) = \phi^c(x) + \delta\phi^c(x)$ , where  $\delta\phi^c(x) = i\alpha \star \phi^c(x)$ , with  $\alpha = \alpha_0$  or  $\alpha = \alpha_1$ , the variation of the action yields the result:

$$\begin{aligned} \delta \mathcal{S}_{\text{YM}} &= \int d^2x \delta((\mathcal{L}_{\text{YM}} \star e^{-1})e) \\ &= \int d^2x \{\delta_{A_\mu}((\mathcal{L}_{\text{YM}} \star e^{-1})e) + \delta_{\phi^c}((\mathcal{L}_{\text{YM}} \star e^{-1})e)\} \\ &= \int_M d^2x \partial_\sigma (\mathcal{J}^\sigma + \mathcal{K}^\sigma) = 0 \end{aligned} \quad (67)$$

Then  $\mathcal{J}^\sigma$  can be computed in the same way as for symmetric  $\omega_{ab}^\mu$ . See relation (49). The gauge invariance of the YM action furnishes the current  $\mathcal{J}'^\sigma = \mathcal{J}^\sigma + \mathcal{K}^\sigma$ . Under vanishing condition of the surface terms,  $\mathcal{J}'^\sigma$  is locally conserved on shell.

## 5. Concluding remarks

In this work, we have defined the twisted connections in noncommutative spaces and discussed NC gauge transformations. Then, the YM action, twisted in the dynamical Moyal space, has been proved to be invariant under  $U_\star(1)$  gauge transformation with the parameter  $\alpha = \alpha_0 + \epsilon_\mu x^\mu$ , where  $\epsilon_\mu$  is an infinitesimal parameter and  $\alpha_0$  a constant. The gauge action is defined in two-dimensional Moyal space with signature (1, 1). The NC gauge-invariant currents are explicitly computed. These currents are locally conserved.

Finally, it is worth mentioning that the approach developed here can be extended to investigate twisted gauge theory in finite  $D$ -dimensional Moyal space. The only technical difficulty resides in the fact that the choice of  $\omega$  could not be arbitrarily made. For this reason, the canonical form of  $e_a^\mu$  given by  $\delta_a^\mu + \omega_{ab}^\mu x^b$  seems to be natural. The trivial case  $\omega = 0$  corresponds to NC YM theory, well known in the literature.

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**Appendix**

Notwithstanding the condition  $[X_a, X_b] \neq 0$ , i.e.  $\omega_{ab}^\mu$  is skew-symmetric, the twisted  $\star$ -product defined in (2) remains noncommutative and associative. Indeed, using the twisted star-product

$$(f \star g)(x) = m \left[ \exp \left( \frac{i}{2} \theta e^{-1} \epsilon^{\mu\nu} \partial_\mu \otimes \partial_\nu \right) (f \otimes g)(x) \right], \tag{A.1}$$

one can see that

$$e^{ikx} \star e^{iqx} = e^{i(k+q)x} e^{-\frac{i}{2} \theta e^{-1} k \epsilon q}. \tag{A.2}$$

The Fourier transform of  $f, g \in \mathcal{S}(\mathbb{R}_0^2)$  can be written as

$$\tilde{f}(k) = \int d^2x e^{-ikx} f(x), \quad \tilde{g}(q) = \int d^2x e^{-iqx} g(x), \tag{A.3}$$

with the function inverse transform given by

$$f(x) = \int d^2k e^{ikx} \tilde{f}(k), \quad g(x) = \int d^2q e^{iqx} \tilde{g}(q). \tag{A.4}$$

We can redefine the twisted star-product of two Schwartz functions  $f, g$  as

$$\begin{aligned} (f \star g)(x) &= \int d^2k d^2q \tilde{f}(k) \tilde{g}(q) e^{ikx} \star e^{iqx} \\ &= \int d^2k d^2q \tilde{f}(k) \tilde{g}(q) e^{i(k+q)x} e^{-\frac{i}{2} \theta e^{-1} k \epsilon q}. \end{aligned} \tag{A.5}$$

Then, we have

$$\begin{aligned} ((f \star g) \star h)(x) &= \left[ \int d^2k d^2q \tilde{f}(k) \tilde{g}(q) e^{-\frac{i}{2} \theta e^{-1} k \epsilon q} e^{i(k+q)x} \right] \star \left[ \int d^2p e^{ipx} \tilde{h}(p) \right] \\ &= \int d^2k d^2q d^2p \tilde{f}(k) \tilde{g}(q) \tilde{h}(p) (e^{-\frac{i}{2} \theta e^{-1} k \epsilon q} e^{i(k+q)x}) \star e^{ipx}. \end{aligned} \tag{A.6}$$

Recalling that  $e^{-1} = 1 + \omega_\mu x^\mu$ , we obtain

$$\begin{aligned} ((f \star g) \star h)(x) &= \int d^2k d^2q d^2p \tilde{f}(k) \tilde{g}(q) \tilde{h}(p) e^{-\frac{i}{2} (\theta k \epsilon q - \frac{1}{2} \theta^2 \omega(k \epsilon q) \epsilon p)} e^{-\frac{i}{2} \theta e^{-1} (k+q) \epsilon p} \\ &\quad \times e^{i(k+q+p - \frac{1}{2} \theta \omega k \epsilon q)x} \\ &= \int d^2k d^2q d^2p \tilde{f}(k) \tilde{g}(q) \tilde{h}(p) e^{-\frac{i}{2} (\theta k \epsilon q + \theta (k+q) \epsilon p - \frac{1}{2} \theta^2 \omega(k \epsilon q) \epsilon p)} \\ &\quad \times e^{i(k+q+p - \frac{1}{2} \theta \omega (k+q) \epsilon p - \frac{1}{2} \theta \omega k \epsilon q)x}. \end{aligned} \tag{A.7}$$

On the other hand,

$$\begin{aligned} (f \star (g \star h))(x) &= \int d^2k d^2q d^2p \tilde{f}(k) \tilde{g}(q) \tilde{h}(p) e^{ikx} \star (e^{-\frac{i}{2} \theta e^{-1} q \epsilon p} e^{i(q+p)x}) \\ &= \int d^2k d^2q d^2p \tilde{f}(k) \tilde{g}(q) \tilde{h}(p) e^{-\frac{i}{2} (\theta q \epsilon p + \theta k \epsilon (q+p) - \frac{1}{2} \theta^2 \omega(k \epsilon q) \epsilon p)} \\ &\quad \times e^{i(k+q+p - \frac{1}{2} \theta \omega q \epsilon p - \frac{1}{2} \theta \omega k \epsilon (q+p))x} \end{aligned} \tag{A.8}$$

A straightforward expansion shows that (A.7) and (A.8) coincide. This results in the conclusion that the used twisted star-product (2) is well associative.

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